Perth Modern School

Question/Answer Booklet

MATHEMATICS:SPECIALIST 3CD

Semester One Exam 2011

Section One: Calculator-free

SOLUTIONS

Section One
(calculator-free)

Time allowed for this section

Section One
Reading time before commencing work: 5 minutes
Working time for paper: 50 minutes

Material required/recommended for this paper

To be provided by the supervisor
Question/answer booklet for Section One, containing a removable formula sheet which may also be used for Section Two.

To be provided by the candidate
Section One:

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this examination

	Number of questions available	Number of questions to be attempted	Suggested working time (minutes)	Marks available
Section One Calculator-free	7	7	**50**	**40**
Section Two Calculator-assumed	10	10	100	80
			Total marks	120

Instructions to candidates

1. The rules for the conduct of WACE external examinations are detailed in the booklet *WACE Examinations Handbook*. Sitting this examination implies that you agree to abide by these rules.

2. Answer the questions in the spaces provided.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

4. Spare answer pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continues i.e. give the page number.

MARK ALLOCATION AND RECORDS:

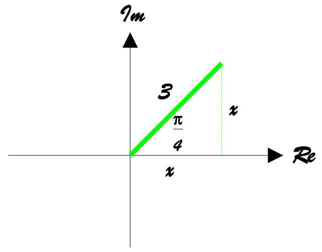
Section	**Question**	**Marks**	**Awarded**
ONE	1	2	
	2	4	
	3	7	
	4	9	
	5	6	
	6	6	
	7	6	
	Penalties	- 1/2/3	
	ONE	**40**	
	TWO	**80**	
	TOTAL	**120**	

Penalties	
Rounding (-1)	
Units (-1)	
Notation (-1)	

%	

Question 1 [2 marks]

Express the polar co-ordinates [3, $\frac{\pi}{4}$] in Cartesian form.



$$\sin\frac{\pi}{4} = \frac{x}{3} = \frac{1}{\sqrt{2}}$$

$$\therefore x = \frac{3}{\sqrt{2}}$$

$$\Rightarrow \left[3, \frac{\pi}{4}\right] = \left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$$ ✓✓

Question 2 [4 marks]

Find the vector equation and Cartesian equation of the plane containing the point A (1,3,-2) perpendicular to the direction vector **n** = 3**i** – 4**j** + 5**k**.

Position vector of A: $a = i + 3j - 2k$ ✓

Vector normal: $n = 3i - 4j + 5k$ ✓

The vector equation of the plane: $(r - a).\ n = 0$

If $r = xi + yj + zk$, then

$$\begin{pmatrix} x-1 \\ y-3 \\ z+2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix} = 0$$ ✓

and the Cartesian equation is $3(x - 1) - 4(y - 3) + 5(z + 2) = 0$

$$\Rightarrow 3x - 4y + 5z + 19 = 0$$ ✓

Question 3 [1, 1, 2, 2, 1 = 7 marks]

For the complex numbers **z** = a + bi and **w** = c + di, where a, b, c, d ∈ **R**, evaluate the following in terms of a, b, c and/or d, as appropriate:

a) $\bar{\mathbf{z}} = a - bi$ ✓

b) $\overline{\mathbf{w}} = c - di$ ✓

c) $\overline{\mathbf{z} \times \mathbf{w}} = \overline{(a + bi) \times (c + di)}$

$= \overline{(ac - bd) + (ad + bc)i}$ ✓

$= (ac - bd) - (ad + bc)i$ ✓

d) $\overline{\mathbf{z}} \times \overline{\mathbf{w}} = (a - bi) \times (c - di)$ ✓

$= (ac - bd) - (ad + bc)i$ ✓

e) What is the relationship between $\overline{\mathbf{z} \times \mathbf{w}}$ and $\overline{\mathbf{z}} \times \overline{\mathbf{w}}$?

$\overline{z \times w} = \overline{z} \times \overline{w}$ ✓

Question 4 [9 marks]

A city has four television channels – W, X, Y and Z. Between 6:00 pm and 6:30 pm, each night, Channels X and Y broadcast the news and the other two channels broadcast a variety of other shows. A survey was conducted to determine the percentage of people watching each channel in the 6:00-6:30 pm timeslot, assuming no-one watched more than one channel.

It was found that 16% of the people surveyed did not watch television at that time and, of those watching television, ¾ were watching the news. The ratio of the number of people watching Channel X to the number of people watching Channel Z was 5:1, while the percentage of people watching Channel X was 21% more than the percentage watching Channel W.

Let w% be the percentage of people watching Channel W,
x% be the percentage of people watching Channel X,
y% be the percentage of people watching Channel Y and
z% be the percentage of people watching Channel Z.

Write equations, in the above four variables, to form a system of linear equations. Write these equations in matrix form so that a solution **could** be found using an inverse matrix **BUT DO NOT SOLVE THIS SYSTEM**.

$w + x + y + z = 84$ ✓

$x + y = 63$ ✓

$w + z = 21$ ✓

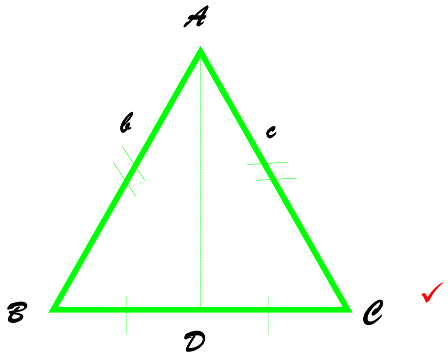
$x = 5z$ ✓

$x = w + 21$ ✓

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -5 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 84 \\ 63 \\ 0 \\ 21 \end{bmatrix}$$ ✓✓✓✓

Question 5 [6 marks]

ΔABC is an isosceles triangle with AB = AC. If D is the midpoint of BC, prove that s∠ADC = 90°.
[**HINT:** First draw a diagram, and let $\overrightarrow{AB}$ = **b** and $\overrightarrow{AC}$ = **c**.]



$\overrightarrow{BC} = c - b$ ✓

$\overrightarrow{BD} = \frac{1}{2}(c - b)$ ✓

$\overrightarrow{AD} = \frac{1}{2}(c + b)$ ✓

$\overrightarrow{AD} \cdot \overrightarrow{BC} = \frac{1}{2}(c + b) \cdot (c - b)$

$= |c|^2 - |b|^2$

But $|b| = |c|$

$\Rightarrow \overrightarrow{AD} \cdot \overrightarrow{BC} = 0$ ✓

$\therefore \overrightarrow{AD} \perp \overrightarrow{BC}$ ✓

Question 6 [6 marks]

The line **r** = (2**i** + 3**j**) + λ(-**i** + 3**j** + 2**k**) passes through A when λ = 1, and B when λ = 5. Find the position vector of the point C where $\overrightarrow{AC}$:$\overrightarrow{BA}$ = -1:4.

$A = i + 6j + 2k$ ✓

$B = -3i + 18j + 10k$ ✓

$BA = \begin{pmatrix} 4 \\ -12 \\ -8 \end{pmatrix}$ ✓

$r_c = \begin{pmatrix} 1 \\ 6 \\ 2 \end{pmatrix} + \left(-\frac{1}{4}\right)\begin{pmatrix} 4 \\ -12 \\ -8 \end{pmatrix}$ ✓

$= \begin{pmatrix} 0 \\ 9 \\ 4 \end{pmatrix}$ ✓

$\therefore C = 9j + 4k$ ✓

Question 7 [1, 1, 1, 1, 1, 1 = 6 marks]

If **z** = 3 cis $\frac{\pi}{4}$ and **w** = 2 cis $\frac{\pi}{3}$. Express the following in polar form:

a) $\frac{\mathbf{z}}{\mathbf{w}} = \frac{3}{2} cis\left(-\frac{\pi}{12}\right)$ ✓

b) $\mathbf{zw} = 6\ cis\left(\frac{7\pi}{12}\right)$ ✓

c) $\bar{\mathbf{z}} = 3\ cis\left(-\frac{\pi}{4}\right)$ ✓

d) $i\mathbf{w} = 2\ cis\left(\frac{5\pi}{6}\right)$ ✓

e) $\mathbf{w}^3 = 8\ cis\ (\pi)$ ✓

f) $\left(\frac{\mathbf{w}}{\mathbf{z}}\right)^6 = \frac{64}{243} cis\left(\frac{\pi}{2}\right)$ ✓

End of questions for section one

SPARE PAGE FOR WORKING

SPARE PAGE FOR WORKING

Perth Modern School

Question/Answer Booklet

MATHEMATICS:SPECIALIST 3CD

Semester One Exam 2011

Section Two: Calculator-assumed

Name ______________________________

Teacher ____________________

Section Two
(calculator-assumed)

Time allowed for this section

Section Two
Reading time before commencing work: 10 minutes
Working time for paper: 100 minutes

Material required/recommended for this paper

To be provided by the supervisor
Question/answer booklet for Section Two. Candidates may use the removable formula sheet from Section One.

To be provided by the candidate
Section Two:

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

Special items: drawing instruments, templates, notes on up to two unfolded sheets of A4 paper, and up to three calculators, CAS, graphic or scientific, which satisfy the conditions set by the Curriculum Council for this course.

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			Total marks	120

Instructions to candidates

1. Answer the questions in the spaces provided.

2. **Show all your working clearly**. Your working should be in sufficient detail to allow your answei to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

3. **For any question or part question worth more than two marks, valid working or justification i required to receive full marks**.

4. Spare answer pages are provided at the end of this booklet. If you need to use them, indicate in th original answer space where the answer is continued i.e. give the page number.

MARK ALLOCATION AND RECORDS:

Section	Question	Marks	Awarded
TWO	8	2	
	9	8	
	10	4	
	11	6	
	12	7	
	13	6	
	14	7	
	15	17	
	16	15	
	17	7	
	Penalties	- 1/2/3	
	ONE	**40**	
	TWO	**80**	
	TOTAL	**120**	

Penalties	
Rounding (-1)	
Units (-1)	
Notation (-1)	

%

Question 8 [2 marks]

Express the Cartesian co-ordinates $(-\sqrt{3}, -1)$ in polar form.

$$\text{ToPol}\begin{bmatrix} -\sqrt{3} \\ -1 \end{bmatrix} \Rightarrow [2, -\frac{5\pi}{6}] \checkmark\checkmark$$

Question 9 [2, 2, 4 = 8 marks]

Given the matrix $M = \begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix}$, calculate:

a) $M^2 = \begin{bmatrix} 5 & -8 \\ -8 & 13 \end{bmatrix}$ ✓✓

b) $M^{-1} = \begin{bmatrix} -3 & -2 \\ -2 & -1 \end{bmatrix}$ ✓✓

c) Using three of the following matrices $\begin{bmatrix} 1 & 2 & -3 \\ -2 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix}$ and $\begin{bmatrix} 4 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$, perform a calculation of the form B(A + C). **Show working to support your answer.**

$$B(A+C) = \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix} \times \left(\begin{bmatrix} 1 & 2 & -3 \\ -2 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} \right) \checkmark\checkmark$$

$$= \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix} \times \begin{bmatrix} 5 & 2 & -4 \\ 0 & 1 & 1 \end{bmatrix} \checkmark$$

$$= \begin{bmatrix} 5 & 0 & -6 \\ 15 & 5 & -13 \end{bmatrix} \checkmark$$

Question 10 [2, 2 = 4 marks]

a) Find exact values for the real and imaginary parts of $\dfrac{4 + 5\mathbf{i}}{2 - 3\mathbf{i}}$.

$$\frac{4 + 5i}{2 - 3i} \times \frac{2 + 3i}{2 + 3i} = \frac{-7 - 22i}{13}$$

$$\text{Real part} = -\frac{7}{13} \quad ✓$$

$$\text{Imaginary part} = -\frac{22}{13} \quad ✓$$

b) Find the modulus and argument of 3 – 4i.

$$\text{Mod} = \sqrt{3^2 + (-4)^2} = 5 \quad ✓$$

$$\text{Arg} = -\tan^{-1}\left(\frac{4}{3}\right) \approx -0.9273 \approx -53.1^\circ \quad ✓$$

Question 11 [6 marks]

Find the centre and radius of the circle in the complex plane whose equation is 2| **z** + 3i | = | **z** – 3 |.

$$2^2 \left| z + 3i \right|^2 = \left| z - 3 \right|^2 \quad ✓$$

$$4[x^2 + (y + 3)^2] = [(x - 3)^2 + y^2]$$

$$4(x^2 + y^2 + 6y + 9) = x^2 - 6x + 9 + y^2$$

$$4x^2 + 4y^2 + 24y + 36 = x^2 - 6x + 9 + y^2 \quad ✓$$

$$3x^2 + 6x + 3y^2 + 24y + 27 = 0$$

$$x^2 + 2x + y^2 + 8y + 9 = 0$$

$$(x + 1)^2 + (y + 4)^2 - 1^2 - 4^2 + 9 = 0 \quad ✓$$

$$(x + 1)^2 (y + 4)^2 = 8 \quad ✓$$

$\therefore$ Circle has centre $(-1, -4)$ or $-1 - 4i$ and radius $\sqrt{8}$ or $2\sqrt{2}$. ✓✓

Question 12 [8 marks]

Consider the pattern of Hexagonal numbers: 1, 6, 15, 28, 45, 66, … which has the recursive formula $T_{n+1} = T_n + (4n + 1)$ and the general formula $T_n = 2n^2 - n$. Use the recursive formula to validate the general formula **using** the method of **induction**.

Let $T = 1 \Rightarrow T_1 = 1$ ← Correct ✓

Let $n = k \Rightarrow T_k = 2k^2 - k$ ← Assumed true ✓

Thus, $T_{k+1} = 2(k+1)^2 - (k+1)$ ✓

$= 2(k^2 + 2k + 1) - k - 1$ ✓

$= 2k^2 + 4k + 2 - k - 1$ ✓

$= (2k^2 - k) + (4k + 1)$ ✓

$= T_k + (4k + 1)$ ✓

Hence, the general formula is correct. ✓

Question 13 [3, 1, 2 = 6 marks]

Immunologists modelled the following situation of the recurrence of seasonal flu in the population.

- 20 % of people who have the virus this year will contract it next year.
- 80% of people who have the virus this year will NOT contract it next year.
- 40% of people who have NOT had the virus this year will contract it next year.
- 60% of people who have NOT had the virus this year will NOT contract it next year.

Given 20 000 out of a population of 100 000 contracted the flu in 2010, use matrices and the above model to determine:

a) how many people will and will NOT contract the flu in 2011?

Let T ~~=~~ This $\begin{bmatrix} 0.2 & 0.4 \\ 0.8 & 0.6 \end{bmatrix}$ ✓ and $M = \begin{bmatrix} 20\,000 \\ 80\,000 \end{bmatrix}$ ✓

$T \times M = \begin{bmatrix} 36\,000 \\ 64\,000 \end{bmatrix}$ ⇒ 36 000 will and 64 000 will not have the flu. ✓

b) how many people will and will NOT contract the flu in 2012?

$T^2 \times M = \begin{bmatrix} 32\,800 \\ 67\,200 \end{bmatrix}$ ⇒ 32 800 will and 67 200 will not have the flu. ✓

c) the long term number of people who will and will NOT contract the flu?

$T^{15} \times M \approx \begin{bmatrix} 33\,333 \\ 66\,667 \end{bmatrix}$ ⇒ In the long term 33 333 will and 66 667 will not have the flu. ✓✓

Question 14 [7 marks]

Take any two-digit number, reverse the digits, and then find the difference between the numbers. Prove, algebraically, that the difference is a multiple of 9.

Let $n = ab$ where a and b are digits 0 to 9 and $a \neq b$, and ✓

$m = ba$ ✓

Thus, $n = 10a + b$ ✓

and $m = 10b + a$ ✓

$\Rightarrow n - m = 9a - 9b$ ✓

$= 9(a - b)$ ✓

$\therefore$ The difference is a multiple of 9. ✓

Question 15 [2, 2, 3, 3, 2, 5 = 17 marks]

Suppose that **z** is a complex number with a modulus of **r** and an argument of θ. Express, in terms of **r** and θ, the modulus and argument of each of the complex numbers $\mathbf{z_1}$, $\mathbf{z_2}$, $\mathbf{z_3}$, $\mathbf{z_4}$ and $\mathbf{z_5}$ where:

a) $\mathbf{z_1} = \overline{\mathbf{z}}$

$\Rightarrow |z_1| = r$ ✓ and $\arg(z_1) = -\theta$ ✓

b) $\mathbf{z_2} = i\mathbf{z}$

$\Rightarrow |z_2| = r$ ✓ and $\arg(z_2) = \theta + \frac{\pi}{2}$ ✓

c) $\mathbf{z_3} = -\mathbf{z}^{-1}$

$\Rightarrow |z_3| = \frac{1}{r}$ ✓ and $\arg(z_3) = -\theta + \pi$ ✓✓

d) $\mathbf{z_4}$ and $\mathbf{z_5}$ are the square roots of **z**.

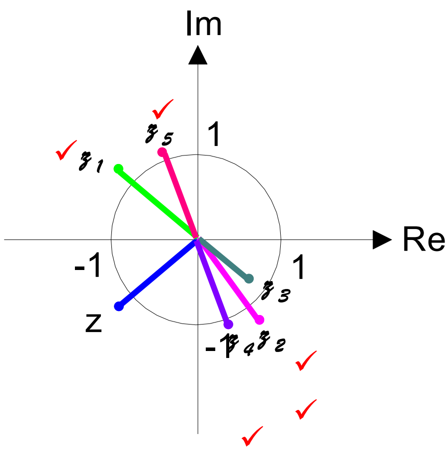
$\Rightarrow |z_4| = |z_5| = \sqrt{r}$ ✓ and $\arg(z_4) = \frac{\theta}{2}$ ✓ $\arg(z_5) = \frac{\theta}{2} + \pi$

✓

This diagram shows the unit circle in the complex plane and the position of the complex number **z**.

e) Estimate the modulus **r** and the argument θ.

$r \approx 1.3$ ✓ and $\theta \approx -140°$ ✓



f) Indicate, as accurately as possible on the diagram, the locations of the complex numbers $\mathbf{z_1} \rightarrow \mathbf{z_5}$ as defined above.

$z_1 \Rightarrow |z| = 1.3$ & $\arg = 140°$

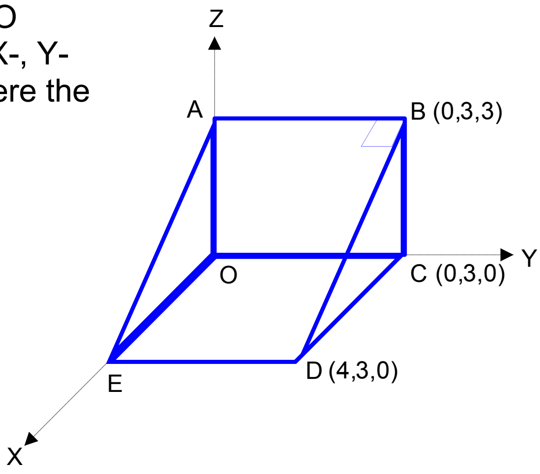
$z_2 \Rightarrow |z| = 1.3$ & $\arg \approx 54°$

$z_3 \Rightarrow |z| \approx 0.77$ & $\arg = -40°$

$z_4 = z_5 \Rightarrow |z| \approx 1.14$ & $\arg = -70°$ & $110°$ [z_4 & z_5 are interchangeable.]

Question 16 [2, 2, 2, 2, 3, 4 = 15 marks]

The figure in the diagram is a triangular prism with its vertex O (0,0,0) and the edges OE, OC and OA are placed along the X-, Y- and Z-axes respectively. The diagram represents a ramp where the units are in metres.



a) Give the co-ordinates of the vertices A and E.

A (0,0,3) ✓ and E (4,0,0) ✓

b) Find the vector and parametric equations of the line $\overline{\mathbf{AD}}$.

$\overrightarrow{OP} = \overrightarrow{OA} + \lambda\overrightarrow{AD}$

Vector equation: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ -3 \end{pmatrix}$ ✓

Parametric equations: $x = 4\lambda$, $y = 3\lambda$, $z = 3 - 3\lambda$ ✓

c) Give the component forms for the vectors $\overline{\mathbf{DB}}$ and $\overline{\mathbf{DC}}$.

$\overrightarrow{DB} = \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 3 \end{pmatrix}$ ✓

$\overrightarrow{DC} = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 0 \end{pmatrix}$ ✓

d) Use your answers in Part c) to calculate s∠BDC.

$\overrightarrow{DB} . \overrightarrow{DC} = \begin{pmatrix} -4 \\ 0 \\ 3 \end{pmatrix} . \begin{pmatrix} -4 \\ 0 \\ 0 \end{pmatrix} = 16$ ✓

$$\cos\theta = \frac{\overrightarrow{DB} . \overrightarrow{DC}}{|\overrightarrow{DB}| \times |\overrightarrow{DC}|}$$

$\theta \approx 36.9°$ ✓

A small trolley is released from a point, M midway between A and B, and travels in a line parallel to $\overline{\mathbf{BD}}$.

e) Give the vector equation of the path followed by the trolley.

$\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} = \begin{pmatrix} 0 \\ 1.5 \\ 3 \end{pmatrix}$ ✓

Vector equation: $\overrightarrow{OR} = \overrightarrow{OM} + \mu\overrightarrow{BD}$ as parallel to $\overrightarrow{BD}$. ✓

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1.5 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$ ✓

f) Assuming the trolley is made to travel at a constant velocity of 0.5 m/s parallel to $\overline{\mathbf{BD}}$, calculate its position in space 5 seconds after it is released.

Let V be velocity vector $\parallel \overrightarrow{BD} \Rightarrow |V| = 0.5$ and $|BD| = 5 \Rightarrow$

$$V = \frac{|V|}{|BD|} \times \overrightarrow{BD} = \frac{1}{10}\overrightarrow{BD}$$ ✓

$$\overrightarrow{OR} = OM + V \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1.5 \\ 3 \end{pmatrix} + 5\begin{pmatrix} 0.4 \\ 0 \\ -0.3 \end{pmatrix}$$ ✓✓

$$\Rightarrow \overrightarrow{OR} = \begin{pmatrix} 2 \\ 1.5 \\ 1.5 \end{pmatrix} \text{ metres}$$ ✓

Question 17 [1, 1, 1, 4 = 7 marks]

Age (years)	0-1	1-2	2-3	3-4	4-5	5-6
Population	2 500	2 000	1 800	1 500	900	500
Birth Rate	0	0	1.5	1.7	1.2	0.5
Survival	0.7	0.75	0.8	0.9	0.3	0

a) Set this information up in a Leslie matrix.

$$L = \begin{bmatrix} 0 & 0 & 1.5 & 1.7 & 1.2 & 0.5 \\ 0.7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.75 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.3 & 0 \end{bmatrix}$$ ✓

b) Set out a matrix to represent the initial sub-groups within the population.

$$H = \begin{bmatrix} 2\,500 \\ 2\,000 \\ 1\,800 \\ 1\,500 \\ 900 \\ 500 \end{bmatrix}$$ ✓

It is proposed that the 2-4 year olds be harvested from the herd.
c) Given the harvesting rate is *h*, rewrite the Leslie matrix to include this new information.

$$\begin{bmatrix} 0 & 0 & 1.5(1-h) & 1.7(1-h) & 1.2 & 0.5 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 0.7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.75 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8(1-h) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9(1-h) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.3 & 0 \end{bmatrix}$$ ✓

d) By investigating the growth of the total population of the herd, determine the best harvesting rate from 25%, 30% or 35%.

$J = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]$

For $h = 0.25$, $J \times L_1^{15} \times H \approx [25\,433]$

$J \times L_1^{16} \times H \approx [27\,172]$ ✓

$J \times L_1^{17} \times H \approx [29\,010]$

For $h = 0.30$, $J \times L_1^{15} \times H \approx [16\,638]$

$J \times L_1^{16} \times H \approx [17\,284]$ ✓

$J \times L_1^{17} \times H \approx [17\,946]$

For $h = 0.35$, $J \times L_1^{15} \times H \approx [10\,578]$

$J \times L_1^{16} \times H \approx [10\,664]$ ✓

$J \times L_1^{17} \times H \approx [10\,746]$

$\therefore$ The best harvesting rate is 35%. ✓

End of questions for section two

SPARE PAGE FOR WORKING

SPARE PAGE FOR WORKING